

# *A Generic Framework for Local Search : Application to the Sudoku Problem*

*ICCS 2006, Reading, UK*

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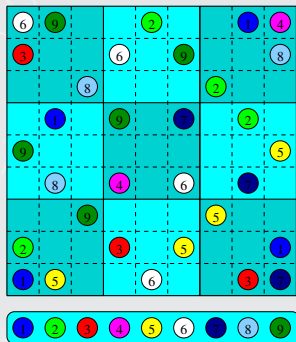
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joint work with E. Monfroy and F. Saubion

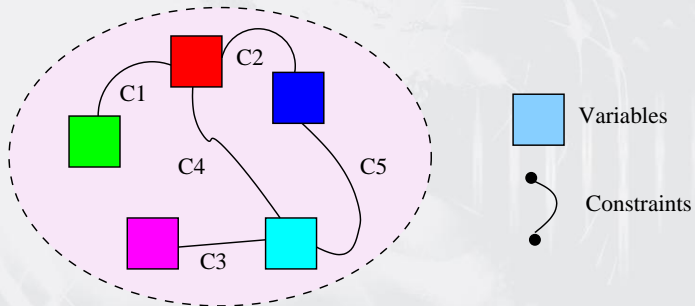
**Wed, 31 May, 2006**

# Outline

- Local search for solving CSP
- A framework for local search techniques
- Generic Iteration Algorithm with reduction functions
- Some results for the sudoku problem
- Conclusion



# *Constraint satisfaction Problem (CSP)*



## *Local search (1)*

### Definition :

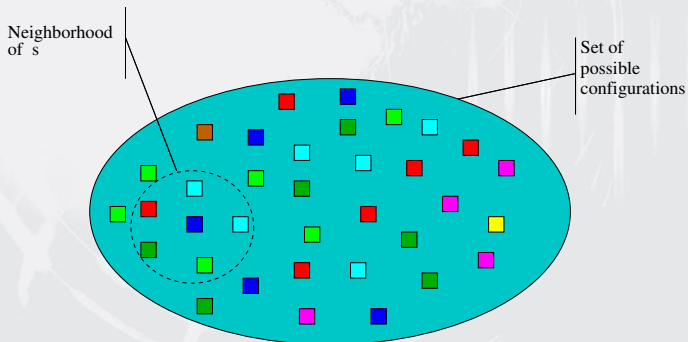
- Explore a  $D_1 \times D_2 \times \dots \times D_n$  search space
- Move from neighbor to neighbor thanks to an evaluation function
  - Intensification
  - Diversification

### Properties :

- focus on some “promising” parts of the search space
- does not answer to unsat. problems
- no guaranteed
- “fast” to find a “good” solution

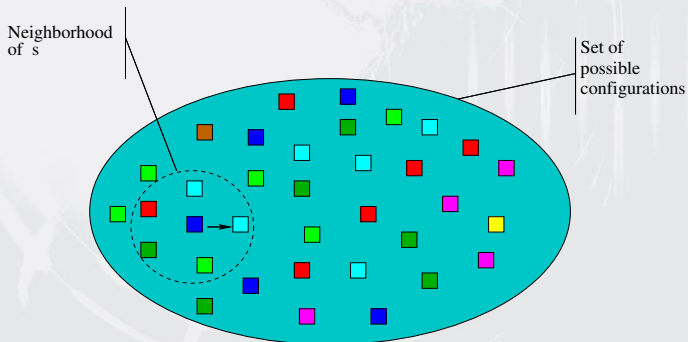
## Local search (2)

- Search space : set of possible configurations
- Tools : neighborhood and evaluation function



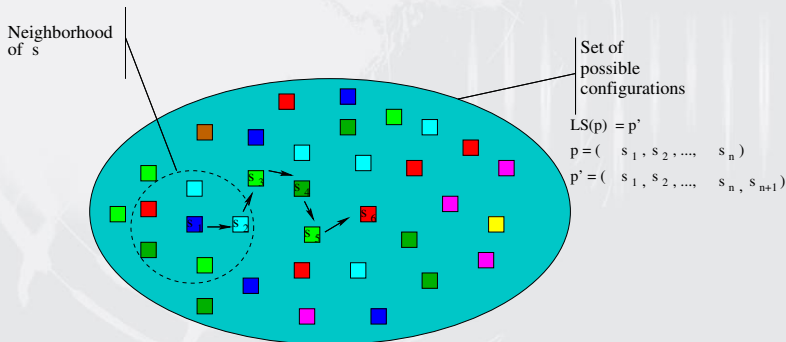
## Local search (3)

- Search space : set of possible configurations
- Tools : neighborhood and evaluation function



## Local search (4)

- Search space : set of possible configurations
- Tools : neighborhood and evaluation function



## *A framework for local search techniques*

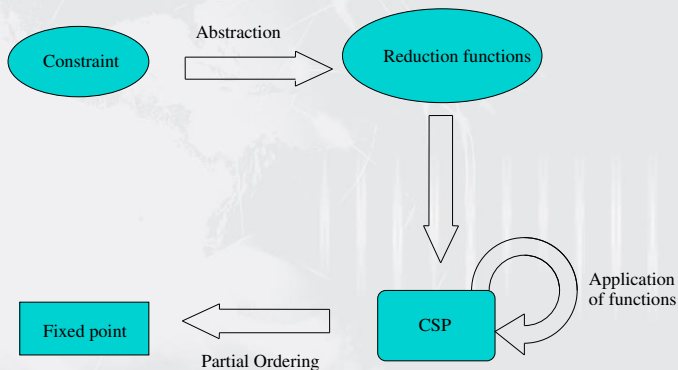
- Idea :
  - Fine grain control
  - More strategies
- Technique :
  - Decomposing local search resolution into basic functions
  - Adapting chaotic iterations



## *Our Purpose :*

- Use of an existing theoretical model for CSP solving
- Definition of the solving process

## *Abstract Model K.R. Apt [CP99]*



## A Generic Algorithm

$F = \{ \text{set reduction functions} \}$

$X = \text{initial CSP}$

$G = F$

While  $G \neq \emptyset$

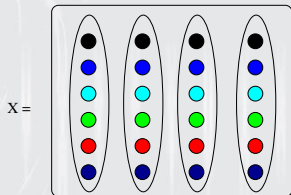
    choose  $g \in G$

$G = G - \{g\}$

$G = G \cup \text{update}(G, g, X)$

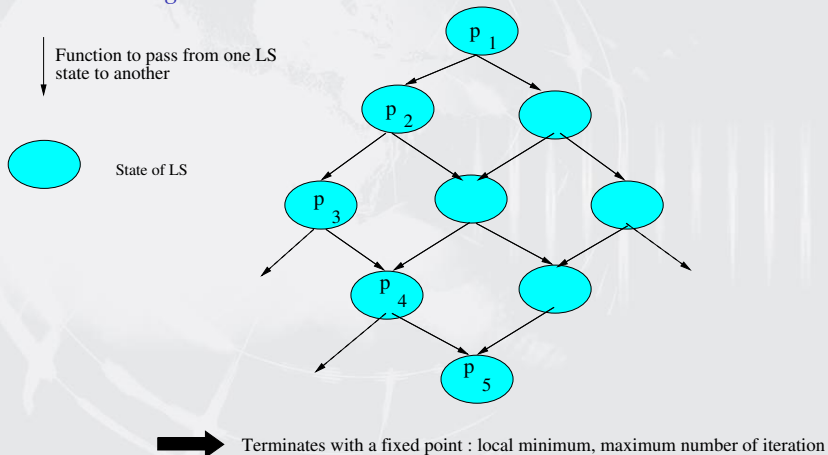
$X = g(X)$

EndWhile



## The theoretical model for CSP solving

*Partial ordering :*



## *Ls Ordering*

### Characteristics of a LS path

- notion of solution
- maximum length

Operational (computational) point of view : path = samples

A sample :

- Depends on a CSP
- Corresponds to a local search state

## Neighborhood and Move functions

Neighborhood :

- FullNeighbor :  $V' = \{s \in D \mid s \notin V\}$
- TabuNeighbor :  $V' = \{s \in D \mid \nexists k, n - l \leq k \leq n, s_k = s\}$
- DescentNeighbor :  $p = (s_1, \dots, s_n)$  and  $V' = s \subset D$  s.t. /  $\exists s' \in V$  s.t  $eval(s') < eval(s_n)$

Move :

- BestMove :  $p' = p \oplus s'$  and  $eval(s') = \min_{s'' \in V} eval(s'')$
- ImproveMove :  $p = p'' \oplus s_n$  and  $p' = p \oplus s$  s.t.  $eval(s') < eval(s_n)$
- RandomMove :  $p' = p \oplus s'$  and  $s' \in V$

## *Instantiate the GI algorithm*

We can precise here the input set of function  $F$ .

Tabu search :

- TabuNeighbor
- BestNeighBor

Random walk + Descent :

- FullNeighbor
- BestNeighBor
- RandomNeighbor
- DescentNeighbor
- ImproveNeighBor

Random walk :

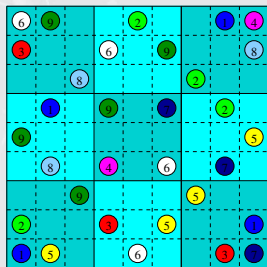
- FullNeighbor
- BestNeighBor
- RandomNeighbor

TabuSearch + Descent :

- TabuNeighbor
- DescentNeighbor
- ImproveNeighBor
- BestNeighBor

## Sudoku problem

- $n^2 \times n^2$  problem
- $n^4$  variables
- *AllDiff* constraints





# *Sudoku problem*

6	9			2		1	4	
3	2	1	6		9		8	
		8				2		1
	1		9		7		2	3
9								5
	8		4		6		7	9
8	3	9				5	6	2
2			3		5			1
1	5			6			3	7

# *Sudoku problem*

6	9	5	7	2	8	3	1	4
3	2	1	6	4	9	7	5	8
7	4	8				2	9	1
4	1		9	5	7		2	3
9								5
5	8		4		6		7	9
8	3	9				5	6	2
2			3	9	5			1
1	5	4		6		9	3	7

# *Sudoku problem*

6	9	5	7	2	8	3	1	4
3	2	1	6	4	9	7	5	8
7	4	8	5			2	9	1
4	1	6	9	5	7	8	2	3
9						6	4	5
5	8		4		6		7	9
8	3	9				5	6	2
2	6	7	3	9	5	4	8	1
1	5	4		6		9	3	7

# *Sudoku problem*

6	9	5	7	2	8	3	1	4
3	2	1	6	4	9	7	5	8
7	4	8	5	1	3	2	9	1
4	1	6	9	5	7	8	2	3
9	7	3	2	8	1	6	4	5
5	8	2	4	3	6	1	7	9
8	3	9	1	7	4	5	6	2
2	6	7	3	9	5	4	8	1
1	5	4	8	6	2	9	3	7

	TabuSearch			RandomWalk		
$n^2 \times n^2$	16x16	25x25	36x36	16x16	25x25	36x36
cpu time	3,14	115,08	3289,8	3,92	105,22	2495
deviations	1,28	52,3	1347,4	1,47	49,3	1099
mvts	405	3240	22333	443	2318	13975
	Descent + TabuSearch			Descent + RandomWalk		
$n^2 \times n^2$	16x16	25x25	36x36	16x16	25x25	36x36
cpu time	2,34	111,81	2948	2,41	82,94	2455
deviations	1,42	55,04	1476	1,11	36,99	1092
mvts Avg	534	3666	20878	544	2581	14908

## *Conclusion and Future works*

- A generic model local search methods
- Design of strategies
- Dynamic strategies learning
- Providing more tools in a generic environment
- A generic implementation

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