Solving Strategies using a Hybridization Model for Local Search and Constraint Propagation

ACM SAC 2005, Santa Fe, New Mexico, USA

Tony Lambert and Éric Monfroy and Frédéric Saubion

LERIA, Université de Angers, France
and
LINA, Université de Nantes, France

March 16, 2005

Outline

- Hybridization
- Constraint propagation
- Local search
- Hybrid model
- Some results
- Conclusion

Hybridization for CSP

- complete methods (such as propagation + split)
  - complete exploration of the search space
  - detects if no solution
  - generally slow for hard combinatorial problems
  - global optimum

Hybridization for CSP

- complete methods (such as propagation + split)
  - complete exploration of the search space
  - detects if no solution
  - generally slow for hard combinatorial problems
  - global optimum
- incomplete methods (such as local search)
  - focus on some “promising” parts of the search space
  - does not answer to unsat. problems
  - no guaranteed global optimum
  - “fast” to find a “good” solution

Hybridization : getting the best of both methods

- But, generally :
  - Ad-hoc systems
  - Master-slaves approaches
  - Coarse grain cooperation

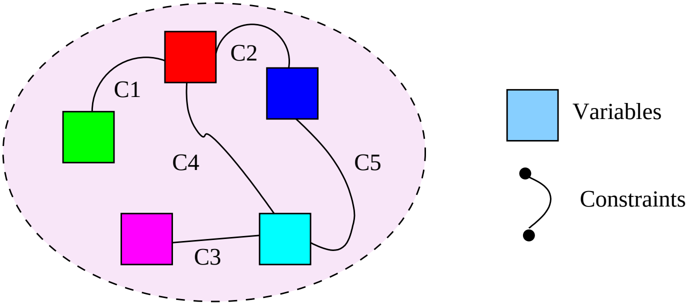
Hybridization : getting the best of both methods

- But, generally :
  - Ad-hoc systems
  - Master-slaves approaches
  - Coarse grain cooperation
- Idea :
  - Fine grain control
  - More strategies

Hybridization : getting the best of both methods

- But, generally :
  - Ad-hoc systems
  - Master-slaves approaches
  - Coarse grain cooperation
- Idea :
  - Fine grain control
  - More strategies
- Technique :
  - Decomposing solvers into basic functions
  - Adapting chaotic iterations for hybrid solving

CSP (Constraint Satisfaction Problem)



Problems are modeled as CSP (X, D, C)

Variable (X)
Set of variable domains (D)

$$D_x = \{a; b; c; \ldots\}$$

$$D_y = \{a; b; c; \ldots\}$$

$$D_z = \{a; b; c; \ldots\}$$

$$\ldots$$

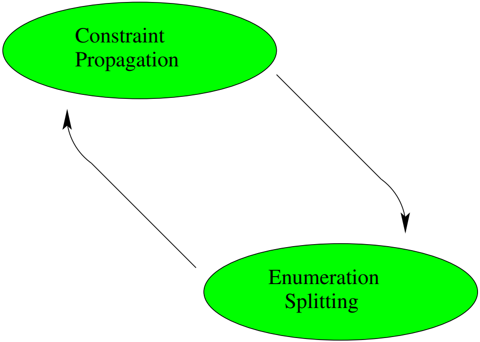
Set of constraints (C)

$$C_1 : X \leq Y * 3$$

$$C_2 : Z \neq X - Y$$

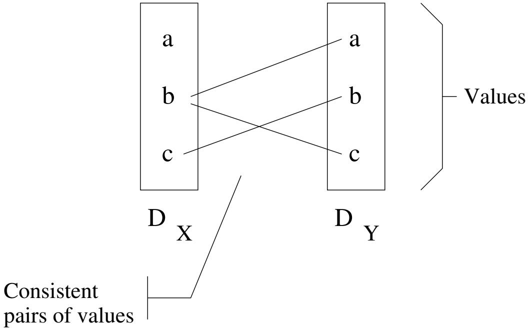
$$\ldots$$

Solving CSP with a complete method



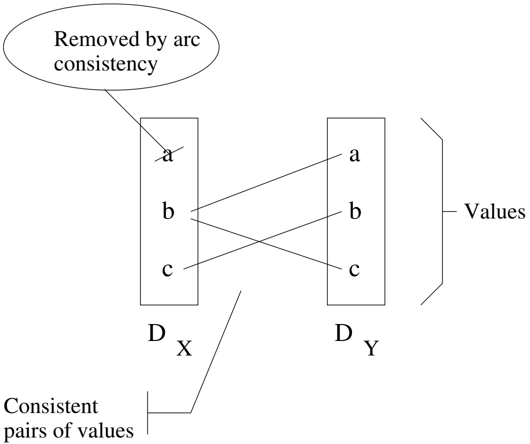
Constraint Propagation

Arc consistency : $C(X, Y)$

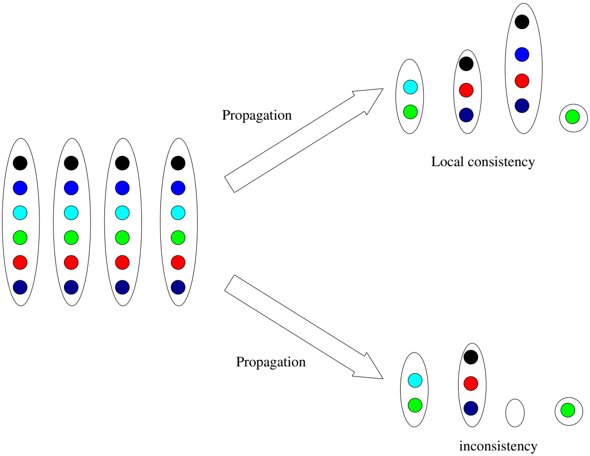


Constraint Propagation

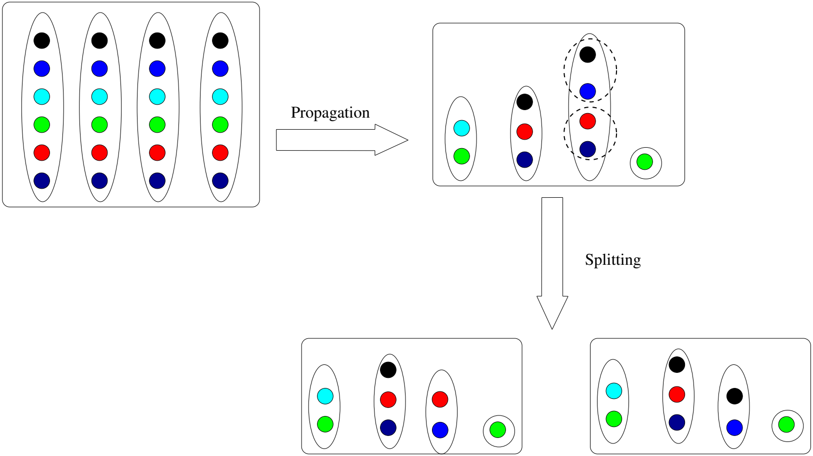
Arc consistency : $C(X, Y)$



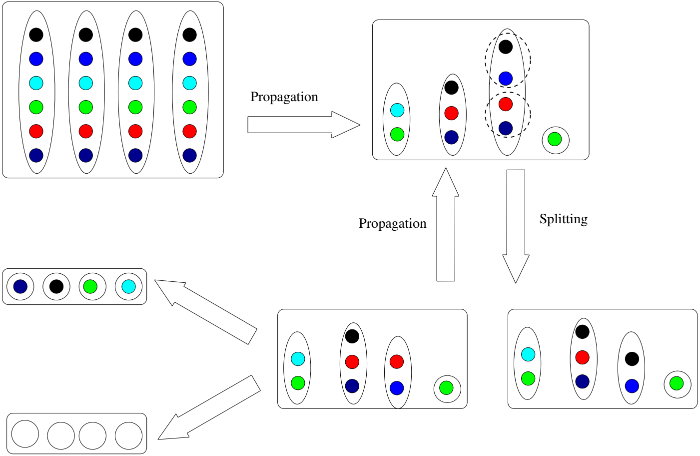
Constraint Propagation



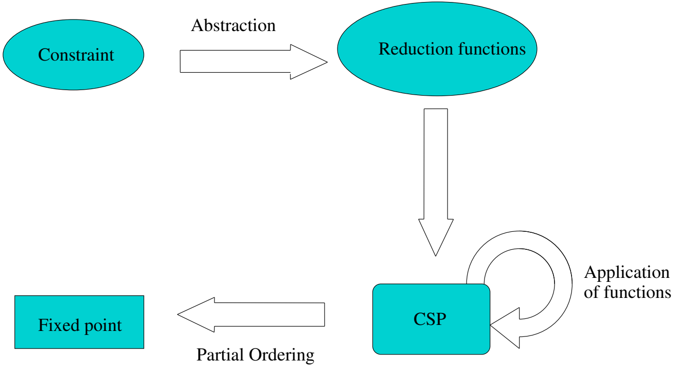
Constraint propagation and domain splitting



Constraint propagation and domain splitting



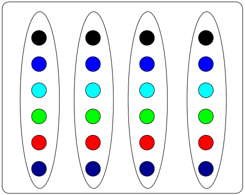
Abstract Model K.R. Apt [CP99]



A Generic Algorithm

$F = \{$ set of split and propagation functions$\}$

$X =$ initial CSP
$G = F$
While $G \neq \emptyset$
    choose $g \in G$
    $G = G - \{g\}$
    $G = G \cup update(G, g, X)$
    $X = g(X)$
EndWhile

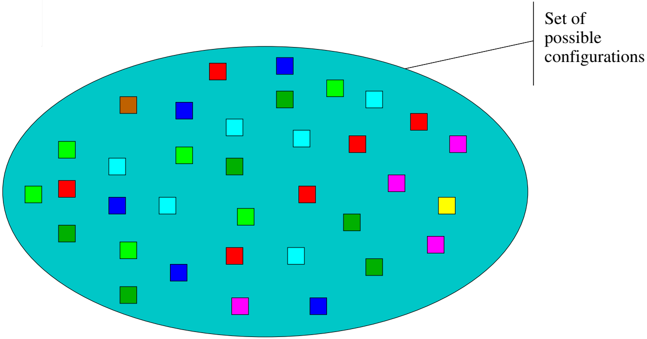
X = 

Our Purpose : Hybrid Model

- Integration of local search
- Use of an existing theoretical model for CSP solving
- Definition of the solving process

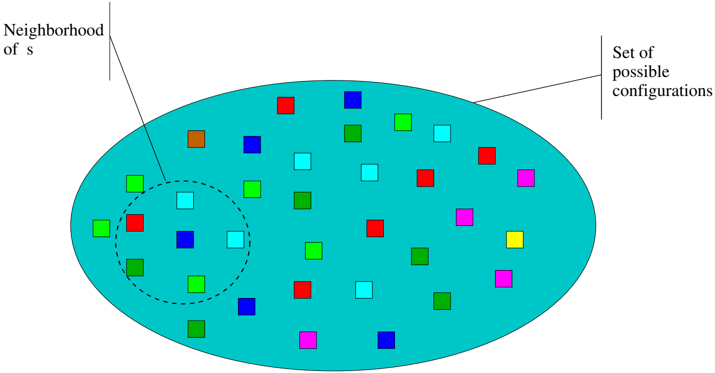
Local search

- Search space : set of possible configurations
- Tools : neighborhood and evaluation function



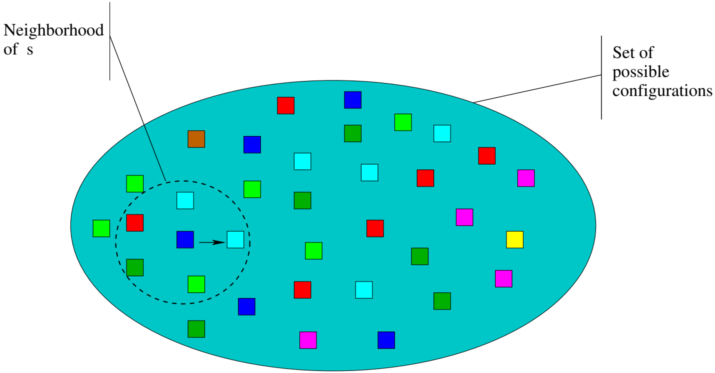
Local search

- Search space : set of possible configurations
- Tools : neighborhood and evaluation function



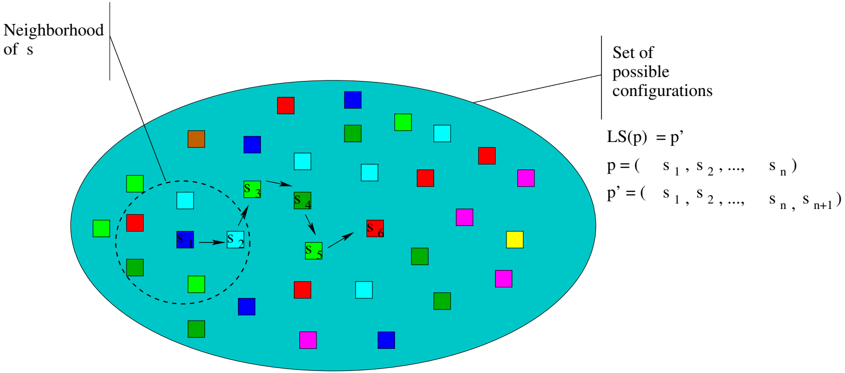
Local search

- Search space : set of possible configurations
- Tools : neighborhood and evaluation function



Local search

- Search space : set of possible configurations
- Tools : neighborhood and evaluation function

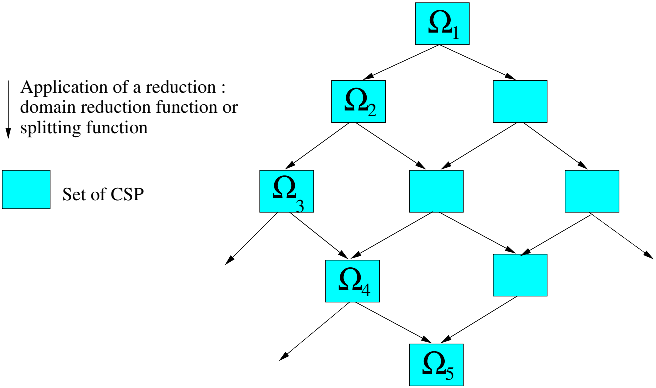


Integrating CSP Resolution Techniques in a Hybrid Model

- Extend the existing Apt ′ s theoretical model for CSP solving
- Fixpoint computation on a partial ordering

The theoretical model for CSP solving

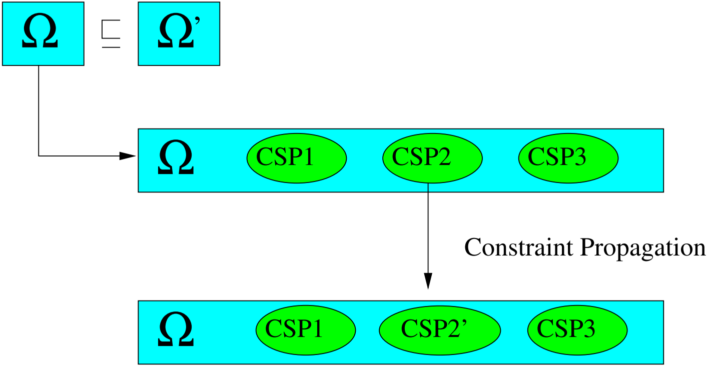
Partial ordering :



Terminates with a fixed point : set of solutions or inconsistent CSP

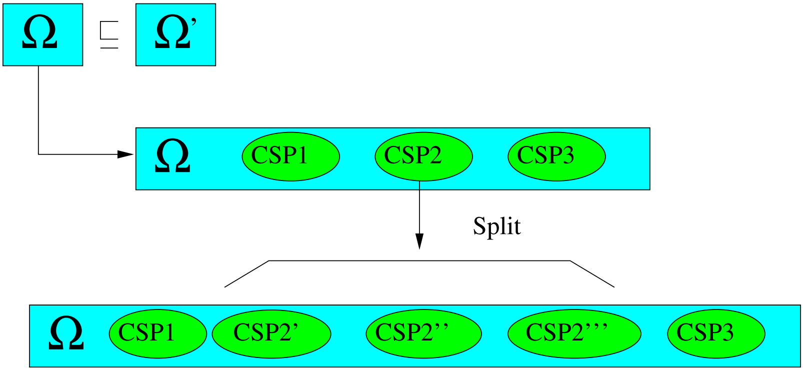
The theoretical model for CSP solving

Reduction : by constraint propagation



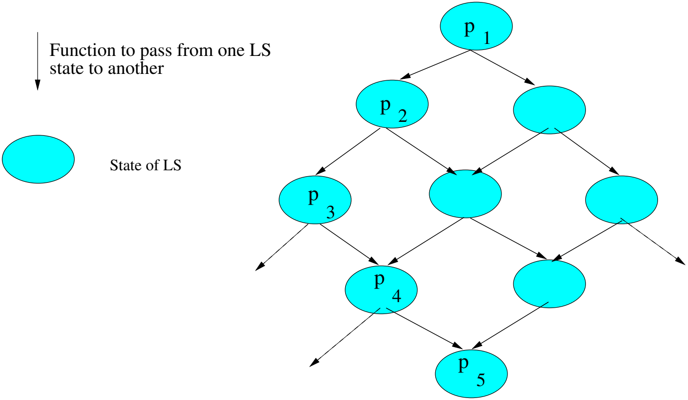
The theoretical model for CSP solving

Reduction : by domain splitting



LS process as moves on partial ordering

Moves on a partial ordering



Terminates with a fixed point : local minimum, maximum number of iteration

LS Ordering

Characteristics of a LS path

- Notion of Solution
- Maximum Length

Operational (computational) point of view : path = samples

Integration of samples for local search into the CSP

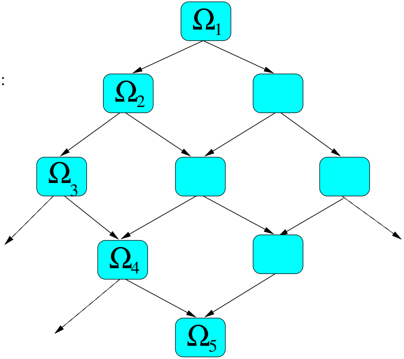
- A sample :
  - Depends on a CSP
  - Corresponds to a LS state
- An SCSP contains :
  - A set of domains ;
  - A set of constraints ;
  - A (list of) sample for local search.

Hybridization model

Ordering over SCSP

Application of a reduction : function :
- domain reduction,
- splitting,
- or LS step.

Set of SCSP



➡ A solution before the end of the process, given by LS

➡ Terminates with a fixed point : set of solutions or inconsistent SCSP

Hybridization model

Ordering over SCSP

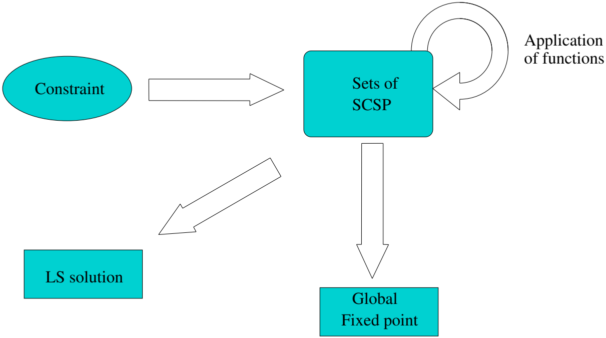
$$\Psi \sqsubseteq \Psi' \quad \text{with :} \quad \Psi = \langle C; D; p \rangle$$
$$\Psi' = \langle C; D'; p' \rangle$$
$$\text{if :} \quad D \subseteq D'$$
$$\text{or if :} \quad D' = D \text{ and } p \sqsubseteq p'$$

Algorithm

Same Generic Algorithm

$X =$ initial $SCSP$
$G = F$
While $G \neq \emptyset$
    choose $g \in G$
    $G = G - \{g\}$
    $G = G \cup update(G, g, X)$
    $X = g(X)$
EndWhile

General Process



Algorithm

- In the chaotic iteration framework
- Finite domains (sets of SCSP)
- Monotonic functions (LS move, domain reduction, splitting)
- Decreasing functions
- $\rightarrow$ termination and computation of fixed point (solution of CP)
- Practically : can stop before with a LS solution

Examples

- SEND + MORE = MONEY
- Zebra
- Golomb
- Magic square
- Langford Number

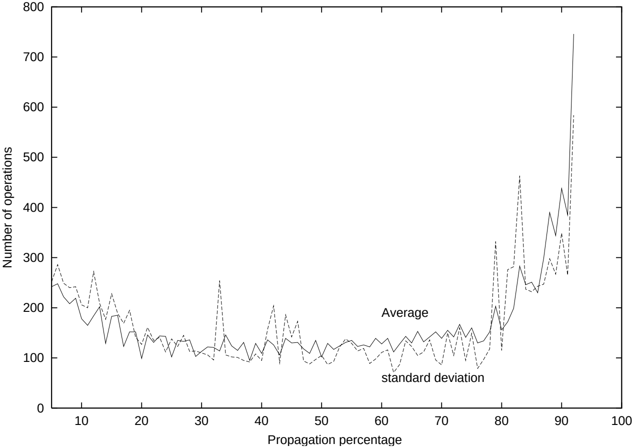
Modeling Strategies through Reduction Functions

- Reduction / Split / LS functions
- Choose a / several SCPS to apply functions
- Choose a / several domains (Prop. - Split)
- Tests : Random - Depth first - FC

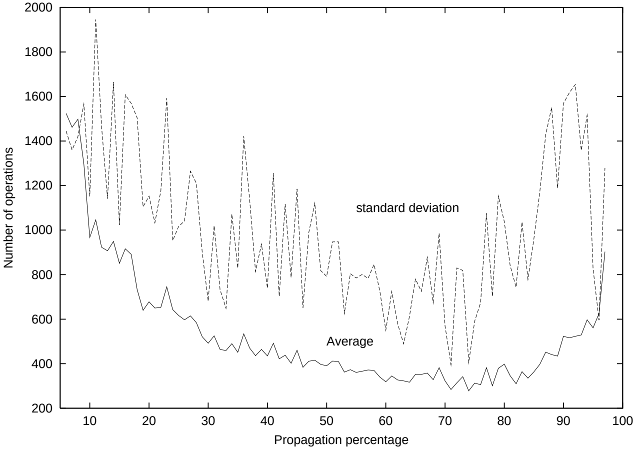
Strategy : ratio LS-CP

- Ratio of LS functions applied and CP functions applied : Triple(red%,split%,ls%)
  - 10% of split compared to reduction
  - CP alone : (90,10,0)
  - LS alone : (0,0,100)
- LS strategy : tabu
- Choose : LS, reduction, or split but keep the given ratios

Langford number 4 2



SEND + MORE = MONEY



Problem	S+M	LN42	Zebra	M. square	Golomb
Rate FC	70-80	15 - 25	60-70	30-45	30 - 40

TAB.: Best range of propagation rate (α) to compute a solution

Strategy	Method	S+M	LN42	M. square	Golomb
Random	LS	1638	383	3328	3442
	CP+LS	1408	113	892	909
	CP	3006	1680	1031	2170
D-First	LS	1535	401	3145	3265
	CP+LS	396	95	814	815
	CP	1515	746	936	1920
FC	LS	1635	393	3240	3585
	CP+LS	22	192	570	622
	CP	530	425	736	1126

TAB.: Avg. nb. of operations to compute a first solution

Conclusion

- A generic model for hybridizing complete (CP) and incomplete (LS) methods
- Implementation of modules working on the same structure
- Complementarity of methods
- Design of strategies

Future work

- Adaptation of the model for optimization
- Study of other methods (G.A.)
- Design of strategies using composition operators
- A generic implementation